PERIODICA
Journal of Modern Philosophy, Social Sciences and Humanities
ISSN NO:2720-4030 Volume 23 October, 2023

Geometric solution of problems involving irrationality

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ABSTRACT

This article presents the solutions of some Olympiad problems on the topic of triangles, which are not often encountered. Through these, the student learns that he can solve Olympiad problems not in the same way, but also through different creative thinking

ARTICLE INFO

Received: 14th August 2023
Revised: 14th September 2023
Accepted: 20th October 2023

KEYWORDS: Triangle, inequality, proof, theorem of cosines, perimeter, bisector, angle. theorem of sines, center of gravity, triangle inequality.

Algebraization of mathematics education, the false belief that we can solve any text problems with the help of equations, led to the almost disappearance of sufficiently complex, but no less beautiful problems from the school mathematics course. At the same time, there are a number of problems, including that it is more convenient to solve "geometric" than "algebraic" in the competitive exam.

The discovery of the existence of irrational numbers shook the foundations of ancient Greek mathematics. (According to legend, Hippasus was expelled from school by the Pythagoreans for his proof - they did not want to believe in the existence of infinite quantities.)

Issue 1. $\sqrt{2}$ prove that the number is irrational

Solution:

Geometrically $\sqrt{2}$ irrational means the diagonal of a square with its sides, or the hypotenuse of an equilateral right triangle: neither there nor here has an integer cutoff. (Fig. 1.) This can be checked as follows: we divide the smaller section into a larger section, put the remainder into the smaller section, the second remainder into the first, and so on. If at any stage the cross-section is left without a residue, then this is the common measure of the two initial cross-sections (Euclidean algorithm). Let's put a section equal to the leg on the hypotenuse. For this, we make a triangle with the bisector of the angle.

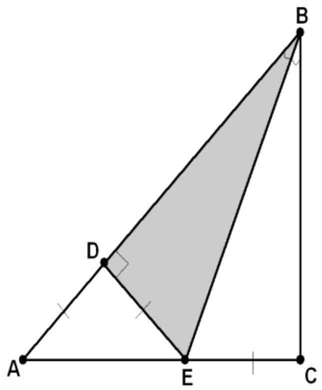

Figure 1

AD a residue appeared. Then we need to put this "residue" in the catheter, but it is already "put" (CE section) it is not difficult to see. New remains ADE enters a triangle - again equilateral and right-angled! Continuing the process, we do the same with the new triangle as we did with the large triangle, resulting in an even smaller triangle, and so on. We get an infinite process with an infinite reduction of the cross section. Thus, the hypotenuse and the hypotenuse are irrational.

A geometric proof is unique - it is the only one $\sqrt{2}$ works for , but this calculation gives it an approximation algorithm.

Problem 2. Find the smallest value of the expression

$\sqrt{(x-9)^2+4}+\sqrt{x^2+y^2}+\sqrt{(y-3)^2+9}$ (1)

Solution: $\sqrt{(x-9)^2+4}$ number in the coordinate plane $A(9;\pm 2)$ and $B(x;0)$ can be considered as the distance between the points. Similarly, $\sqrt{x^2+y^2}$ son $B(x;0)$ and $C(0;y)$ distance between points $\sqrt{(y-3)^2+9}$ number too $C(0;y)$ and $D(\pm 3;3)$ distance between points.

Thus, expression (1). $AB+BC+CD$ looks like this $ABCD$ the length of the broken line. Only if it is true will the broken line reach its minimum value.

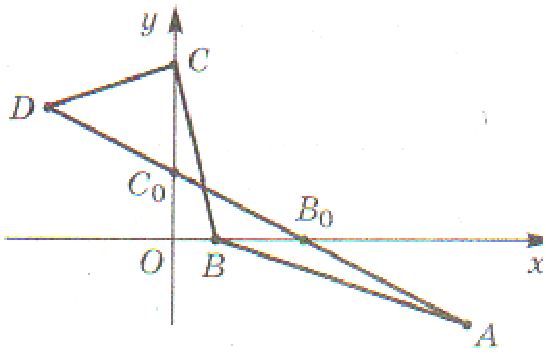

Figure 2

B point Ox because it is relevant and $ABCD$ a straight line, D point II chorakka be relevant and $(-3;3)$ should have coordinates. Likewise, C point Oy since it belongs to A point IV to belong to the quarter and $(9;-2)$ should have coordinates.

In that case $AD=\sqrt{(9+3)^2+(3+3)^2}=13$

AD we determine the equation of a straight line:

$$\begin{cases} 2=-9k+m \\ -3=3k+m \end{cases}$$

$$k=-\frac{5}{12}$$

$$m=\frac{7}{4}$$

$$y=-\frac{5}{12}x+\frac{7}{4}$$

$B\left(x_0;0\right)\in AD$ va $CD\left(0;y_0\right)\in AD$

From this $x_0=\frac{21}{5}, y_0=\frac{7}{4}$

Answer:13; to this value $\left(\frac{21}{5};\frac{7}{4}\right)$ reaches at the point.

Problem 3. Prove:

$$\sqrt{a^2+c^2-ac}\geq\sqrt{a^2+b^2}-\sqrt{b^2+c^2-\sqrt{3}bc}\ .$$

Solution: $\sqrt{(a-\frac{c}{2})^2+(\frac{\sqrt{3}c}{2})^2}+\sqrt{(\frac{c}{2})^2+(b-\frac{\sqrt{3}c}{2})^2}\geq\sqrt{a^2+b^2}$

$\sqrt{(a-\frac{c}{2})^2+(\frac{\sqrt{3}c}{2})^2}$ number in the coordinate plane $A\left(a;0\right)$ va $B(\frac{c}{2};\frac{\sqrt{3}c}{2})$ can be seen as the distance between the points. Similarly,

$\sqrt{(\frac{c}{2})^2+(b-\frac{\sqrt{3}c}{2})^2}$ thigh $B\left(\frac{c}{2};\frac{\sqrt{3}c}{2}\right)$ and $C\left(0;b\right)$, $\sqrt{a^2+b^2}$ number $C\left(0;b\right)$ and $A\left(a;0\right)$ can be seen as the distance between the points.

Thus, the expression (1) looks like this $AB+BC+CA$ ie ABC the perimeter of the triangle. According to the triangle inequality:

$$AB+BC\geq AC$$

Then $\sqrt{(a-\frac{c}{2})^2+(\frac{\sqrt{3}c}{2})^2}+\sqrt{(\frac{c}{2})^2+(b-\frac{\sqrt{3}c}{2})^2}\geq\sqrt{a^2+b^2}$

$$\sqrt{a^2+c^2-ac}\geq\sqrt{a^2+b^2}-\sqrt{b^2+c^2-\sqrt{3}bc}$$

The proof is over.

Problem 4.

$$\begin{cases} 2^{2-x}=4y\sqrt{2} \\ \sqrt{x^2+y^2+1-2x}+\sqrt{x^2+y^2-6x-2y+10}=\sqrt{5} \end{cases}$$

Solution: $\frac{4}{2^x}=4y\sqrt{2}$ (1) $y=\frac{1}{2^{x+1/2}}$ $,t=-x-\frac{1}{2}$ let it be $2^t=-t-\frac{1}{2}$

The first function is increasing, and the second is decreasing. That is, this equation can have only one root. $t=-2$ it is not difficult to notice that the number matches. Then

$$x=\frac{3}{2}, y=\frac{1}{4}.$$

We express the second equation as follows: $\sqrt{(x-1)^2+y^2}+\sqrt{(x-3)^2+(y-1)^2}=\sqrt{5}$

$\sqrt{(x-1)^2+y^2}$ number in the coordinate plane $A(1;0)$ and $B(x;y)$ can be considered as the distance between the points. Likewise, $\sqrt{(x-3)^2+(y-1)^2}$ number too $B(x;y)$ va $C(3;1)$ can be calculated as the distance between the points. And so, (2) expression $AB+BC=AC$ appears, that is $B\in AC$. AC equation of a straight line:

$$\begin{cases} 0=k+m \\ 1=3k+m \end{cases}$$

$$k=\frac{1}{2}$$

$$m=-\frac{1}{2}$$

$$y=\frac{1}{2}x-\frac{1}{2}$$

By substituting the values found in the first equation into this equation, we get the true equation. that is why,

$x=\frac{3}{2}, y=\frac{1}{4}$ numbers are solutions to this equation.

Answer: $\left(\frac{3}{2};\frac{1}{4}\right)$

Issue 5.

$$\begin{cases} x^2+y^2-14x-10y+58 \\ \sqrt{x^2+y^2-16x-12y+100}+\sqrt{x^2+y^2+4x-20y+104}=2\sqrt{29} \end{cases}$$

Solution: $\sqrt{(x-8)^2+(y-6)^2}$ (2) number in the coordinate plane $A(8;6)$ va $B(x;y)$ can be considered as the distance between the points. Similarly, $\sqrt{(x+2)^2+(y-10)^2}$ number $B(x;y)$ and $C(2;10)$ can be calculated as the distance between the points. And so, (2) expression $AB+BC=AC$ appears, that is $B\in AC$.
Then $-2\le x\le 8$ and $6\le y\le 10$

AC equation of a straight line:

$$\begin{cases} 6=8k+m \\ 10=-2k+m \end{cases}$$

$$k=-\frac{2}{5}$$

$$m=\frac{46}{5}$$

$$y=-\frac{2}{5}x+\frac{46}{5}$$

$$(x-7)^2+(y-5)^2=16 \quad (1)$$

$$(x-7)^2+(-\frac{2}{5}x+\frac{46}{5}-5)^2=16$$

This equation has two roots:

$$x=\frac{217-5\sqrt{415}}{29} \text{ yoki } x=\frac{217+5\sqrt{415}}{29}$$

The second root does not satisfy the constraint.

Then $$y=\frac{180+2\sqrt{415}}{29}$$

Answer: $\left(\frac{217-5\sqrt{415}}{29};\frac{180+2\sqrt{415}}{29}\right)$

Problem 6. Find the smallest value of the expression:

$$\sqrt{x^2+y^2}+\sqrt{(x-9)^2+36}+\sqrt{(y-3)^2+9} \quad (1)$$

Solution: $\sqrt{(x-9)^2+36}$ number in the coordinate plane $A(9;\pm 6)$ va $B(x;0)$ can be considered as the distance between the points. Similarly, $\sqrt{x^2+y^2}$ number too $B(x;0)$ va $C(0;y)$ as the distance between points, $\sqrt{(y-3)^2+9}$ and the thigh $C(0;y)$ and $D(\pm 3;3)$ can be calculated as the distance between points.

And so, (1) expression $AB+BC+CD$ it seems $ABCD$ the length of the broken line. If it remains a straight line, it will accept the smallest value of length. B point Ox because it is relevant and $ABCD$ a straight line, D point II to belong to the quarter and $(-3;3)$ should have coordinates. Similarly, C point Oy since it belongs to A point IV to belong to the quarter and $(9;-6)$ should have coordinates.

Then $$AD=\sqrt{(9+3)^2+(3+6)^2}=15$$

Answer: 15.

Examples for independent work

1. Solve the system of equations.

2. $$\begin{cases} 3x+y=9 \\ \sqrt{(x-4)^2+(y+7)^2}+\sqrt{(x-10)^2+(y-1)^2}=10 \end{cases}$$

2. Solve the system of equations.

$$\begin{cases} x^2+y^2-14x-10y+58=0 \\ \sqrt{x^2+y^2-16x-12y+100}+\sqrt{x^2+y^2+4x-20y+104}=2\sqrt{29} \end{cases}$$

3. Solve the system of equations. $\begin{cases} x^2+y^2-14x-10y+58=0 \\ y=-\frac{2}{5}x+\frac{46}{5} \end{cases}$

4. Solve the system of equations.

$$\begin{cases} x+y=1 \\ \sqrt{(x-5)^2+(y-1)^2}+\sqrt{(x+1)^2+(y+7)^2}=10 \end{cases}$$

5. Solve the system of equations

$$. \begin{cases} 5x+3y=2 \\ \sqrt{(x-4)^2+(y+2)^2}+\sqrt{(x-1)^2+(y-2)^2}=5 \end{cases}$$

6. Solve the system of equations.

$$\begin{cases} 2x^3+2y^3+z^3=3 \\ x^6+y^6+z^6=1 \end{cases}$$

7. Solve the system of equations.

$$\begin{cases} x^4+y^4+z^4=1 \\ x^2+y^2-2z^2=2\sqrt{2} \end{cases}$$

8. Solve the system of equations.

$$\begin{cases} 2x+2y=11 \\ \sqrt{(x-3)^2+(y+1)^2}+\sqrt{(x-7)^2+(y-2)^2}=5 \end{cases}$$

9. Solve the system of equations

$$. \begin{cases} 4x-6y=7 \\ \sqrt{(x-7)^2+(y-6)^2}+\sqrt{(x-1)^2+(y+2)^2}=10 \end{cases}$$

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