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Boolean Two Valued Logic: Its Implications To Epistemology

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A B S T R A C T

Binary system of Arithmetic had its Philosophical foundation in ancient China but it was later developed by Leibniz whose consciousness of the system was rekindled by his correspondences with Father Jaochim. In his contributions to the intellectual debate that arose in 1848 that catapulted into the relation of mathematics to logic, Boole attempted to reduce logical processes into arithmetic operations by adopting the binary system values of 0 and 1 wherein he asserted that Logic only trains the mind but does not give us knowledge of reality. What is the epistemic implication(s) of Boole's two valued logic? Can certainty of human knowledge be ascertained in such circumstance? Does Logic give us any knowledge about reality? This article titled: Booleans two valued Logic: its implications to Epistemology adopts the philosophical methods of analysis, criticality and exposition in its attempt to argue that contrary to Boole's argument, two valued Logic guarantees certainty of knowledge as the negation of the false value (or one proposition) implies the certainty of the other (the true value).

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Introduction

The binary system of arithmetic had its philosophical foundation in the work of Gottfried Wilhelm Leibniz (1646 – 1716) published originally as "Explication de L' Arithmetique Binaire" in the *journal of Memories de L' Academic Royale* in 1703 translated as "An Explanation of Binary Arithmetic using only the characters of 0 and 1". Leibnitz discovered the model for this new arithmetic in the five millennia-old book the *Yijing or Zhou Yi or Wade Giles, I-Ching* is at the heart of Chinese Philosophy. When the debate for the reduction of Logic to mathematics and conversely, the reduction of Mathematics to logic raged in the 19th century (Dennis, 2020), George Boole, through his Logical calculus decided to develop the algebra of logic without any reference to numbers" (Jevons, 1874) by working with the values of 0 and 1 having stripped Logic of any epistemic value. Adopting philosophical tools of analysis, exposition and criticality as its arsenals, this article undertakes an excursion of re-interrogating Boolean assertion that Logic only trains the mind in an effort to reincorporate one of the roles of Logic inadvertently relegated by Boole into the corpus of the licensed roles of Logic.

The Philosophical Foundation of Arithmetic

The core questions starring us on the face in this section are: what is the Philosophical foundation of arithmetic? Who was the first to extrapolate algebra into logic?

In the 17th century, the *Yijing* (1 Ching), or Book of changes which is one of the five classics" of Chinese culture that is regarded as the oldest preserved document of Chinese Philosophical thought revived in China was also known in Europe. Gottfried Wilhelm Leibniz (1703) in recognition of the greatness of the Chinese culture asserted that the Occidental and Chinese culture were complementary. Leibniz learned about the Neo-Confucian Shao Yong who had arranged the 64 Hexagrams in precise deductive system.

"Following his interest in numerology, Shao Yong (1011 – 1070) had the conviction that the divine order of the universe could be expressed numerically". Leibniz' correspondences with Father Joachim Bouvet (1650 – 17030) in China exposed Leibniz to the knowledge of Shao Yong's Hexagram that motivated his (Leibniz) publication of "*Explication de L' Arithmétique Binare*" in 1703 translated as "An Explanation of Binary Arithmetic using only the characters 0 and 1. This work first appeared in the *Journal Memories de L' Academic Royale des Science.* Leibniz's article is regarded as the origin of the Binary system. Leibniz's paper contained detailed explanations of how to carry out the four basic arithmetical operations in the binary system having observed that the eight Trigrams of the Yijing were based on two different types of lines on dual system: the divided lines (yin) correlate with 0 (even) and the undivided lines (yang) with 1 (odd).

In the Binary system, the eight Trigrams can represent the values 0 to 63 in a binary code; there is no evidence that Shao Yong already used this numeral system for arithmetical operations such as addition and subtraction. From the above, it is obvious that the binary system was invented by Leibniz. Lawhead (2002) observes that Leibniz had thought that the universe is written in mathematical ideal which makes him to search for universal logically perfect language. Asserting the invention of the binary, Lawhead (2002) writes; "he (Leibniz) eventually developed the system of binary mathematics in which all the numbers may be expressed as combinations of 1's and 0's.

Russell and Whitehead (1910–1913) in the *Principia Mathematica* attempted Logcism, in the *Tractatus,* they attempted to reduce language to a series of propositions that has correspondence with observable facts while Descartes attempted to introduce geometrical and arithmetic methods into Philosophy because of their precisions. Boole (1854) seemed to have this insight when he says "whence it is that the ultimate laws of logic are mathematical in their form"… Leibniz had earlier anticipated the same method and that led to his work on Binary system published in (1703). It is clear from the above that after Leibniz' invention of the binary system, no logician prior to Boole extrapolated it into logic. Therefore, the use of the term "Boolean Algebra" to refer to the introduction of arithmetic into logic is not an act of intellectual crime.

Boolean Algebraic Notation

Boolean Logical Calculus has infinity with arithmetical algebra but it is remarkably different by the index law. Introducing his algebraic notation; Boole (1854) writes in *The Laws of Thought:*

> Now in common Algebra the combination xx is more briefly represented by x^2. Let us adopt same principle of notation here; for the mode of expressing a particular succession of mental operations is a thing in itself quite as arbitrary as the mode of expression a single idea or operation. In accordance with this notation, then the above equation assumes the form $x^2 = x$.

Stating the conditions for his reliance on this, he says:

> All operations of language, as an instrument of reasoning, may be conducted by a system of signs composed of the following elements, 1st Literal symbols, as x, y, etc …, representing things as subjects of our conceptions. 2nd signs of operations, as +, –, x, standing for those operations of mind by which the conceptions of things are

> combined or resolved so as to form new conceptions involving the same elements 3rd The sign of identity.

This shows that Boole based his algebraic notation on a method that is dependent on three rudimentary ideas, to wit: "the the conception of the symbols, the laws of logic formulated as rules for operations upon these symbols, and the consideration that these rules of operation are analogous to those of an algebra of the numbers 0 and 1" (Tall, 2002).

Development of Logical Functions

Boole (1854), defines a function in *The Laws of Thought* thus:

> "Any algebraic expression involving a symbol x is termed a function of x, and may be represented under the abbreviated general form f(x)". Any expression involving two symbols, x and y is similarly, termed a function of x and y, and may be represented under the general form f (x, y) …. .

He also defines the procedure of development of a function when he opines in the same workthat:

> Any function f (x), in which x is a logical symbol, or a symbol of quantity susceptible only of the values 0 and 1 is said to be developed, when it is reduced to the form ax + b (1 – x), a and b being so determined as to make the result equivalent to the function from which it was derived.

The import of the above definition is that it concerns the development of any function; whether logical function or not; in so far as the variable x is restricted to the algebraic values of 1 and 0. Following this conception, therefore, the general formula for development of a logical function could be formulated as:

$$f(x) = ax + b(1 - x).$$

From the above, we can ascertain the values of 'a' and 'b' by substituting for x the values 1 and 0. Then, we have:

$$f(1) = a1 + b(1 - 1) = a$$
$$f(0) = a0 + b(1 - 0) = b$$

If we further substitute in the function, f (f) for 'a' and 'f' (0) for b, then, we can conclude that:

$$f(x) = f(1)x + f(0)(1 - x)$$

This formula is the development of the function f (x) with regard to x.

Boole continues the development of a function involving any member of logical symbols. He starts with a function concerning two symbols, x and y: f (x, y). Considering this first as a function of x alone, he develops it by the general theorem;

$$f(x) = f(1)x + f(0)(1 - x),$$

Boole (1854) notes in *The Laws of Thought* that:

$$f(x, y) = f(1 - y)x + f(0, y)(1 - x).$$

Concerning the result as a function of 'y', Boole (1854), writes:

$$f(x, y) = f(1, 1)xy + f(1, 0)x(1 - y) + f(0, 1)(1 - x)y + f(0, 0)(1 - x)(1 - y),$$

that if the absolute expansion of f (x – y). Therefore, functions involving any number of logical symbols could also be developed through the same procedure or way. The general rule for development of functions is stated by Boole (1854) in *The Laws of Thought* thus:

> 1st to expand any function of the symbols x, y, z – form a series of constituents in the following manner: Let the first constituent be the product of the symbols; change in this product any symbol z into 1 – z, for the second constituent. Then in both these change any other symbol y into 1 – y, for two more constituents. Then in the four constituents thus obtained change any other symbol x into 1 – x, for four new constituents, and so on until the number of possible changes is exhausted. Secondly, to find the coefficient of any constituent – if that constituent involves x as a factor, change in the original function x into 1; but if it involves 1 – x as a factor, change in the original function x into 0. Apply the same rule with reference to the symbol y, z, & c.: the final calculated value of the function thus transformed will be the coefficient sought.

From the above, the general rule of development consists of two parts; the first introduces the constituents of the expansion; the second determines their respective coefficients. Therefore, a function with three arguments has $2^3 = 8$ terms. These terms are formed by a coefficient equal to 0 and 1 and a product of the symbols, that is, the constituent. The sum of the constituents, multiplied each by its respective coefficient, is the development required. For instance, a function involving three logical symbols has the following constituents and coefficients:

xyz
xy (1 – z)
x (1 – y)z
x (1 – y) (1 – z)
(1 – x) yz
(1 – x) y (1 – z)
(1 – x) (1 – y) z
(1 – x) (1 – y) (1 – z)
1, 1, 1
1, 1, 0
1, 0, 1
1, 0, 0
0, 1, 1
0, 1, 0
0, 0, 1
0, 0, 0

Thus, the required development of the function:

$$\begin{aligned} f(x, y, z) = {} & f(1, 1, 1)\, xyz + f(1,1,0)\, xy(1 - z) \\ & + f(1, 0, 1)\, x(1 - y) z + f(1, 0, 0)\, x(1 - y)(1 - z) \\ & + f(0, 1, 1)\, (1 - x) yz + f(0, 1, 0)\, (1 - x) y (1 - z) \\ & + f(0, 0, 1)\, (1 - x)(1 - y) z + f(0, 0, 0)\, (1 - x)(1 - y)(1 - z) \end{aligned}$$

According to Tall (2002), Boole's procedure of development corresponds to disjunctive normal form'. Defining a disjunctive normal form, Tall, (2002) writes: "A formula of the propositional calculus is said to be in disjunctive normal form if it contains disjunction and negation as propositional connectives, applies only to propositional variables, and does not conjoin and disjunction.

In the *Analysis*, Boole (1847) asserts that any formula Φ(x) of propositional logic containing the propositional variable x is equal to the formula:

$$\Phi(1)\, x + \Phi(0)\, (1 - x).$$

0 and 1 are constants of propositional logic for true and false. The quantities Φ(0), Φ(1) are called the moduli of the function Φ(x). Replacing Boole's exclusive disjunction with the inclusive U, the formula, according to Tall (2002), is written in modern notation as,

$$(\Phi(1) \text{ n } x) \text{ U } (\Phi(0) \text{ n } \neg x)$$

in which the disjunctions are incompatible. It follows that Boole was aware that every formula of propositional logic is equal to a disjunctive normal form.

Boole (1854) in *The Laws of Thought* states the conditions of valid reasoning by the aid of symbols, thus:

> 1st, that a fixed interpretation be assigned to the symbols employed in the expression of the data … the laws of the combination of those symbols be correctly determined from that interpretation.
> 2nd, that the formal processes of solution or demonstration be conducted throughout in obedience to all the laws determined above, without regards to the question of the interpretability of the particular results obtained.
> 3rd, that the final result be interpretable in form, and that it be actually interpreted in accordance with that system of interpretation which has been employed in the expression of the data.

Boole's Inconsistent Treatment of the Symbol for Disjunction 'v'

Our focus and particularity in this section is on the inconsistent treatment meted on the conventional symbol for disjunction ‘v’ by Boole. Poetically, a Poet enjoys literal license, whereas in mathematical logic, symbols have constant meanings but in his algebra of logic, Boole introduces such significant and obvious departure in the treatment of the symbol of disjunction ‘v’ that it seems to be rebelling against or conflicting with the well known tradition in which the symbol ‘v’ represented disjunction.

Stating his position concerning the logical connective v in the *On Interpretation*, Aristotle says; … “Everything must either be or not be, whether in the present or in the future … Expressing his Anarchistic conception about symbolic license, Boole (1847) in *The Mathematical Analysis of Logic* writes; “… the validity of the processes of analysis does not depend upon the interpretation of the symbols which are employed, but solely upon the laws of their combination”. Carnap (1934) says the same thing in a different way when he holds that “in logic, there are no morals”; “everyone is at liberty to build up his own logic, i.e. his own language, as he wishes. Perhaps, Putman (1992) succinctly describes Boole’s system of symbol when he observes; “in fact, Boole was quite conscious of the idea of disinterpretation, of the idea of using a mathematical system as an algorithm, transforming the signs purely mechanically without any reliance on meanings”.

In the exercise of his “symbolic liberty”, Boole (1854) in *The Laws of Thought* employs the symbol ‘v’ to represent or express ‘some’ instead of “disjunction” and to this the article submits in correspondence with Tall (2002), that it is confusing. This is also the believe of Jevons (1874) who observes that Boole’s way of expressing the affirmative universal proposition by using the symbol ‘v’ causes complexity, strangles the aesthetics and bareness universality that could have been maintained with fidelity to the ceremonious use of the symbol ‘v’”.

Considering the universal affirmative proposition, “All men are mortal as meaning that “All men are some mortal beings’, Boole (1854) in *The Laws of Thought* employs the symbol v in his expression of some, thus; $y = vx$

This further depicts Boole’s usage of the symbol ‘v’ as that which represents a non-empty class of indefinite extension (Universal class). He also holds that the proposition has an existential import in so far as the symbol ‘v’ is not absolute and could have some of its members … mortal beings. Thus, vx stands for “some x’ or x is not empty. But, this device for the expression of existential propositions is in practice, as noted by Tall (2002) eliminated by Boole – through his (Boole’s) procedure of elimination which consists of substituting 1 and 0 for v, thus; $vy = v$

Following this, the existential import of the universal affirmative is discarded.

With regard to particular affirmative ‘some x’s are y’s

Concerning the particular affirmative proposition; ‘some x’s are y’s, that is written as:

$vx = vy$

Boole (1854) in *The Laws of Thought* employs ‘v’ as the symbol of a class infinite in all ramifications, that it contains some individuals of the class whose expression it is prefixed.

Therefore ‘v’ depends on what it is prefixed to, it is therefore not the same as a class symbol. Boole (1854) states further in *The Laws of Thought* that… ‘v’ is not quite arbitrary, and therefore must not be eliminated as ‘v’ is the representative of some, which, though it may include in its meaning all, does not include none”.

Thus, he accords an unfaithful treatment to the symbol ‘v’ as characterized in some of the instances which include;

Universal affirmative $y = vx$. Inconsistent with the one in the particular affirmative $vx = xy$. As noted by Tall (2002), Boole discards the existential import of ‘v’ from the first expression, but leaves it as it stands in the second one …. Boole’s use of the indefinite class symbol ‘v’ is inconsistent with respect to existential import”. Further, he expresses the particular negative some x’s are not y’s as $vx = v(1 - y)$,

In this expression, as noted by Tall (2002), Boole adopts ‘v’ to represent the indefinite class some, Perice (1870) regards as absurdity since the transposition of $vy = v(1 - x)$

gives us some y’s are not x’s but it does not follow from some x’s are not y’s that some y’s are not x’s. Tall (2002) agrees with Perice (1870) that this expression is therefore wrong and we do not hold any dissenting idea.

Another lack of focus and particularity that is obvious in Boole's treatment of the symbol 'v' is in the expression; vx = vy expressing some men are married and vx = v (1- y) expressing some men are unmarried which is obtained by substitution vy = v (1 – y) expressing some married are unmarried and we agreed with Tall (2002) that it is a contradiction.

Epistemic Imports of Boolean Two-Valued Logic

It is trite that the curiosity to know is the basis for the search for knowledge. This triggered the debate in mathematics between Hamilton and De Morgan concerning the quantification of terms which ignited the consciousness of thinkers concerning the relations of mathematics to Logic (Dennis, 2020). This debate awoke in Boole the desire to contribute to this intellectual debate and he did. Boole (1847) asserts that Logic which trains the mind should form part of Mathematics instead of Philosophy as the former and not the later has relation with numbers. It is not an over statement to assert that Boole was the first to extrapolate the algebraic system from mathematics into Logic, adopts and works with the binary digits of 0 and 1 which represent true and false respectively which was first introduced by Leibniz in 1703.

Concerning the meaning of logical symbols, Boole works without fidelity to pre-established or pre-existing meaning of such symbols having enjoys what we refer to as symbolic license. His position that what matters is the law and not the interpretation of the meaning of the symbols used enjoys concurring opinions from Carnap.

The consequences arising from the proliferations of symbols in logic as noted in the section on Boole's inconsistent treatment of the symbol for disjunction 'v' is that it is capable of creating confusion. As noted by Jevons (1864) Boole's submission that x + x = x is useless.

Despite the numerous criticisms against Boolean two valued logic which is outside the scope of this section to expose, there was some positive epistemological implication(s) inherent in it.

Subsequent to Boole's adoption and working with these binary digits, there were two logical values; true and false. This points to the epistemic implications of working with only two values – true and false as will be shown in this section.

The celebrated tripartite conditions for knowledge put forward by Plato were that of justification, truthfulness and being believable. These held sway until reinterogated by Gettier in his 1963 famous 1963 article, "Is Justified True Belief Knowledge"?. Sextus Empiricus could also be seen as having championed the course for certainty of knowledge and to him and his associates, skepticism means I know that I know nothing instead of I cannot know something but considers knowledge from the perspective of humanizing it. This conception transcends the Cartesian foundation and therefore not susceptible to the problem of infinite regress.

As this section attempts a resume' of the interrogative; what is/are the epistemic imports of Boolean two valued logic? With the aim of showing that Boolean two value logic increases the applicability and reliability of knowledge claim. According to Copi, Cohen and McMahon (2012), there are "nineteen rules of inference" …. Out of these, it is our persuasion that exclusive disjunctive syllogism has affinity with Boolean two valued logic. By way of clarification, disjunction is defines as "an alternation of two statement formed by inserting the word "or" between them . There are two types of disjunction; the weak or inclusive and the strong or exclusive. Inclusive (and/or) implies that at least one of two propositions are true were as exclusive ("X or" implies exactly one must be true, but both cannot be. The symbol for inclusive disjunction is 'v' while '□' is that of the exclusive. At this juncture, an example may explain our point better, thus;

Uyo is the capital of Abia or Akwa Ibom State – p.
Uyo is not the capital of Abia State – q.
Therefore, Uyo is the capital of Akwa Ibom State – q.

Symbolically, this could be represented as;

P □ q
¬ p
:. q

Alternatively, this implies;

P or q

Not p
:. q

In the logic of proposition, disjunctive syllogism is a valid rule of inference. Professing the validity of disjunctive syllogism, the significant of this proposition is clear because at least one of the two sentences is true while the other is certainly false. Boolean two valued logic employs the duo algebraic values of 0 and 1 which represent true and false or affirmation and negation respectively. This implies that given two propositions or situations, the negation of one invariably means the true of the other. This is the basis upon which the ignition key of our car, the electric switch on our wall works. Deducing from the foregoing, it is not tantamount to an act of intellectual dishonesty or an hyperbole to assert, contrary to Boole's view that logic generally and two valued logic in particular does not only train the mind but it guarantees certainty of knowledge as the negation or falsity of one statement invariably means the order is true. Our knowledge in such circumstance is capable of being characterized as certain and objective.

Evaluation

An examination of Boolean two valued logic revealed that he subscribes to the position that Logic trains the mind and that he further believes in symbolic license as evidenced in his infidelity to the usage of the symbol for disjunctive syllogism 'v' as the meaning of symbols to him, is inconsequential. As noted by the controversy between Hamilton and De Morgan about the quantification of predicate serves as the external influence to Boolean algebraic of logic (Tall, 2002, Dennis, 2014). In his contribution to this intellectual furore which was exhumed in the second half of the nineteen century (Tall, 2002), Boole became the first to extrapolate the algebraic method from Mathematics into Philosophy, which in the view was based on the radical application of Ockham's razor.

Conclusion

This research work sets out to examine and expose the epistemic import of Boolean two valued logic and having investigated the philosophical foundation of arithmetic and the background to Boolean two valued logic, it has been found out that it was a child of necessary actuated by the debate concerning the quantification of predicate between Hamilton and De Morgan. A critical perusal of the major and relevant works by Boole concerning his two valued logic also disclosed his conception that logic has as a sole function of training the mind as well as his conviction concerning symbolic license which warranted his inconsistent treatment of the symbol for disjunctive syllogism. The above characterization begs the critical, legitimate and fundamental question; what is the epistemological implication of Boolean two valued logic? The two values he worked with as the builder of the bridge between traditional and modern logic were 0 and 1 otherwise known as true and false. This invariably creates an inseparable affinity between his two valued logic and one of the celebrated rules of inference known as exclusive disjunctive syllogism which is characterized by the notion that given two instances or propositions, the negation or falsity of one implies that the other is necessarily true. This is explicit on the ground that either of the two values must be true while the other is false. There is no middle cause and no possibility of both being true or false. This forms the basis for the numerous applications of the idea of Boolean two valued logic in computer science as its foundation, in the electric switch on the wall of the head of my department, in the ignition key of his car as well as other real life situations. Based on the foregoing, this research submits contrary to Boole's position that logic does not only train the mind but guarantees certainty of knowledge and these forms.

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