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$\tilde{L}_2^{(2)}(0,1]$ Fazoda Optimal Kvadratur Formula Xatolik Funksionali Normasi

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A B S T R A C T

Maqolada, Radon, Furye almashtirishlari va orqaga proyeksiya formulasidan foydalanish va fazoda optimal kvadratur formula xatolik funksionali normasi haqida so’z yuritildi. Furye almashtirishlari fan va texnikada, xususan, kompyuter tomografiyasi (KT) muammolarida keng qo'llaniladi.

A R T I C L E I N F O

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K E Y W O R D S: Furye almashtirishlari, kompyuter tomografiyasi, Radon almashtirishi, kvadratur formulalar

Kirish

Ma’lumki, to'liq uzluksiz rentgen ma'lumotlari mavjud bo'lganda, KT tasvirlari turli xil analitik formulalar yordamida aniq qayta tiklanishi mumkin, masalan, filtrlangan orqa proyeksiyalash formulasi [1-4].
Bunda Radon, Furye almashtirishlari va orqaga proyeksiya formulasidan foydalaniladi. Amaliyotda bizda Radon almashtirishining cheklangan diskret qiymatlari mavjud bo'lgani uchun, biz KT ning filtrlangan orqaga proyeksiyalash usuli bo`yicha Furye integralini taxminan hisoblashimiz kerak bo'ladi. Demak, biz

$$I(f,\omega)=\int_a^b e^{2\pi i\omega x} f(x)dx, \qquad \omega\in\mathbb{R} \qquad (1)$$

Ko`rinishidagi integrallarini taqribiy qiymatini hisoblashimiz kerak.
ω-parametrning katta qiymatlari uchun bunday integrallar *kuchli tebranuvchi* deb nomlanadi. Bunday integrallarni hisoblash maxsus samarali sonli usullarni ishlab chiqishni talab qiladi. Integralni hisoblashning bunday birinchi usuli Fileon tomonidan taklif qilingan [5-9]. Bundan tashqari, kuchli tebranuvchi integrallarga ega bo'lgan integrallarni hisoblashni har xil turlari uchun ko`plab maxsus samarali usullar ishlab chiqilgan.
$(m-1)$-tartibli hosilasigacha absalyut uzluksiz va m- tartibli umumlashgan hosilasi kvadrati bilan [a, b] kesmada integrallanuvchi kompleks qiymatli funksiyalarni ko'rib chiqamiz. $L_2^{(m)}[a,b]$ fazosida skalyar ko`paytma quyidagicha kiritiladi [10-18].

$$\langle f,g\rangle_m=\int_a^b f^{(m)}(x)\bar{g}^{(m)}(x)dx$$

va unga mos keladigan norma quyidagicha aniqlanadi

$$\|f\|_{L_2^{(m)}[a,b]}=\sqrt{\langle f,f\rangle_m}.$$

Biz asosan $T=b-a$ davrli davriy, kompleks qiymatli bo`lgan funksiyalarga mos keladigan $\tilde{L}_2^{(m)}[a,b]$ Sobolev fazosini qaraymiz.
Bu shuni anglatadiki, bu fazodagi har bir element T davriy, ya'ni u bir-biridan o`zgarmas songa farq qiladigan funksiyalar sinfidir.
Shu bilan birga, $L_2^{(m)}[a,b]$ Sobolev fazosida (1) integral integralni sonli hisoblash uchun optimal kvadratur formulalari qurilgan va olingan kvadratur formulalar KT tasvirlarini qayta tiklashda foydalaniladi.
ω ning butun qiymatlarda Furye koeffitsientlari uchun optimal kvadratur formulalaridan foydalangan holda, haqiqiy ω uchun (1) integralni sonli hisoblash uchun approksimatsion formulalar olingan. Keyin ushbu approksimatsion formulalar $\tilde{L}_2(0,1]$ kompleks qiymatli, davriy funksiyalarning fazosida KT tasvirlarini taqribiy qayta tiklash uchun qo'llaniladi [19-24].
Ushbu ishning maqsadi kompleks qiymatli davriy funksiyalarning $L_2^{(2)}$-fazosida ω haqiqiy qiymatlarida (1) integralni taqribiy hisoblash uchun optimal kvadratur formulasini qurish masalasini qarab chiqamiz. Furye almashtirishin aniq funksiyalarda taqribiy hisoblashda olingan optimal kvadratur formulalarni qo'llaymiz.
f funksiya $\tilde{L}_2^{(2)}(0,1]$ Sobolev fazosiga tegishli bo'lsin. $\tilde{L}_2^{(2)}(0,1]$ - Hilbert fazosi kompleks qiymatli, davriy funksiyalarning ikkinchi tartibli umumlashgan hosilalasi kvadrati bilan integrallanuvchi funksiyalar fazosi [25-37].
Bu fazoda f va g ikki funksiyaning skalyar ko`paytmasi quyidagicha aniqlanadi

$$\langle f,g\rangle_2=\int_0^1 f''(x)\bar{g}''(x)dx.$$

Bilamizki, $\tilde{L}_2^{(2)}(0,1]$ fazoda norma quyidagi ko`rinishda bo`ladi

$$\|f\|_{\tilde{L}_2^{(2)}(0,1]}=\sqrt{\langle f,f\rangle_2}.$$

Biz ushbu kvadratur formulani qaraymiz

$$\int_0^1 e^{2\pi i\omega x}f(x)dx\cong\sum_{k=1}^N C_k f(hk), \qquad (2)$$

Bu yerda, $\omega\in R$ va $\omega\neq 0$, $C_k=V_k+iW_k$ ($k=1,2,...,N$) – (2.2) kvadratur formulaning noma'lum koeffitsientlari, $i^2=-1$, $h=1/N$, $N\in\mathbb{N}$.
Integral va yig`indini orasidagi ayirma (2) kvadratur formulaning xatoligi deyiladi va bu xatolikka ushbu chiziqli funksional mos keladi.

$$\ell(x)=\left(\varepsilon_{(0,1]}(x)e^{2\pi i\omega x}-\sum_{k=1}^N C_k\delta(x-hk)\right)*\phi_0(x), \qquad (3)$$

Bu yerda $\varepsilon_{(0,1]}(x)$-$(0,1]$ kesmaning xarakteristik funksiyasi, $\delta(x)$- Dirakning delta funksiyasi, $\phi_0(x)=\sum_{\beta=-\infty}^{\infty}\delta(x-\beta)$ ga teng.

ℓ funksionalni f dagi qiymati (2) kvadratur formulasini xatoligini beradi, ya'ni

$$\ell(f)=(\ell,f)=\int_0^1 e^{2\pi i\omega x}f(x)dx-\sum_{k=1}^N G_k f(hk)=\int_0^1\ell(x)f(x)dx. \qquad (4)$$

(4) xatolik funksionali normasi yuqoridan quyidagicha baholanadi

$$|(\ell,f)|\leq\|\ell\|_{\tilde{L}_2^{(2)*}(0,1]}\|f\|_{\tilde{L}_2^{(2)}(0,1]},$$

Bu yerda $\tilde{L}_2^{(2)*}(0,1]$-fazo $\tilde{L}_2^{(2)}(0,1]$ fazosiga qo`shma fazo.
(2) kvadratur formulaning ℓ xatolik funksionali uchun $\tilde{L}_2^{(2)}(0,1]$ fazoda quyidagi shart o`rinli

$(\ell,1)=0$.

Bu shart (2) kavadratur formulani ixtiyoriy o`zgarmas songa aniq ekanligini va quyidagi tenglik o`rinli ekanligini bildiradi [38-47]

$$\sum_{k=1}^{N} C_k = \int_0^1 e^{2\pi i\omega x}dx. \qquad (5)$$

Ma'lumki, C_k koeffitsientlari (2) kvadratur formulasining o`zgaruvchi parametrlari bo`ladi. (2) kvadratur formulaning xatolik funksionali normasi kvadratini N tugun nuqtalar fiksirlangan holda C_k koeffitsiyentlar bo`yicha minimumga erishtirish natijasida olingan formula $\tilde{L}_2^{(2)}(0,1]$ fazoda optimal kvadratur formula deyiladi.

Ushbu ishda, biz $\tilde{L}_2^{(2)}(0,1]$ davriy, kompleks qiymatli funksiyalar fazosida (2) ko`rinishdagi optimal kvadratur formula quramiz.

Bu yerda biz diskret argument funksiyalari va ular ularning xossalaridan foydalanamiz. $f(hk)$ funksiya diskret argumentli funksiya deyiladi agar u k o`zgaruvchining butun qiymatlari to`plamlar aniqlangan bo`lsa, bu yerda h kichik musbat parametrdir.

$f(hk)$ va $g(hk)$ diskret argumentli funksiyalar svyortkasi quyidagicha aniqlanadi

$$f(hk)*g(hk)=\sum_{l=-\infty}^{\infty} f(hl)g(hk-hl).$$

Bundan tashqari, bizga d^4/dx^4 differentsial operatorning $D_2(hk)$ diskret analogi quyidagi ko`rinishda bo`ladi

$$D_2(hk)=\frac{6}{h^4}\begin{cases} A_1 q_1^{|k|-1}, & \text{agar } |k|\geq 2,\\ 1+A_1, & \text{agar } |k|=1,\\ -8+\dfrac{A_1}{q_1}, & \text{agar } k=0, \end{cases} \qquad (6)$$

Bu yerda $A_1=\dfrac{(1-q_1)^5}{E_3(q_1)}$, $E_3(x)=x^3+11x^2+11x+1$ uchinchi darajali Eyler-Frobenius ko`phadi, $q_1=\sqrt{3}-2$, h -kichik musbat parametr.

Shuni ta`kidlash kerakki, d^4/dx^4 differensial operatorning $D_2(hk)$ diskret analogi qurilgan.

Bu $D_m(hk)$ diskret operatorning $m=2$ bo`lgandagi xususiy holi $D_2(hk)$ dikret operator quyidagi xossalarga ega

$$hD_2(hk)*(hk)^{\alpha}=0, \alpha=0,1,2,3, \qquad (7)$$

$$hD_2(hk)*B_4(kh)=\Phi(hk)-h, \qquad (8)$$

Bu yerda

$$B_4(x)=\sum_{\beta\neq 0}\frac{e^{-2\pi i\beta x}}{(2\pi i\beta)^4} \qquad (9)$$

4-darajali davriy Bernulli ko`phadi, $\Phi(hk)=\sum_{\gamma=-\infty}^{\infty}\delta_d(hk-\gamma)$ va $\delta_d(hk-\gamma)=\begin{cases}1, & hk-\gamma=0,\\ 0, & hk-\gamma\neq 0.\end{cases}$

2. Ekstremal funksiya va xatolik funksiyonali normasi kvadrati

Funksiya $u(x)\in \tilde{L}_2^{(2)}(0,1]$ ekstremal funksiya deyiladi, agar quyidagi tenglikni qanoatlantirsa
$\left(\ell,u\right)=\left\|\ell\right\|_{\tilde{L}_2^{(2)}}\cdot\left\|u\right\|_{\tilde{L}_2^{(2)}}$.

Shunday qilib, $\tilde{L}_2^{(2)}(0,1]$ fazo Hilbert fazosi bo`ladi. Riss teoremasidan (Hilbert fazosida chiziqli funksionallarning umumiy ko`rinishi haqida) xatolik funksionali $\ell(x)$ uchun quyidagi tengligidan o`rinli

$$(\ell, f)=<\psi_\ell, f> \qquad (10)$$

$$\int_0^1 \ell\left(x\right)f\left(x\right)dx=\int_0^1\left(-1\right)^2\bar{\psi}_\ell^{(4)}\left(x\right)f\left(x\right)dx \ .$$

Bu yerdan, $\psi_\ell\left(x\right)$ – uchun quyidagi tenglamani olamiz

$$\bar{\psi}_\ell^{(4)}(x)=(-1)^2\ell(x) \qquad (11)$$

Quyidagi Lemma o`rinli.

Lemma 1. *Ixtiyoriy integrallanuvchi funksiyalar uchun (2) kvadratur formulaning (3) xatolik funksionali uchun ekstremal funksiyasi quyidagicha aniqlanadi*

$$\psi_\ell(x)=(-1)^2\left(\int_0^1 e^{-2\pi i\omega t}\cdot B_4(x-t)dt-\sum_{k=1}^{N}\bar{C}_k\cdot B_4(x-hk)+d_0\right) \qquad (12)$$

Bu yerda d_0 *–* $\forall$ *- o`zgarmas,* $e^{-2\pi i\omega x}$ *va* $\bar{C}_k$ *-* $e^{-2\pi i\omega x}$ *va* $C_k$ *larning kompleks qo`shmasi, o`z navbatida,* $B_4(x)$ *- Bernulli ko`phadi.*

Isbot. (11) tenglamani davriy yechimini topamiz. (12) tenglamada tenglikni har ikki tarafiga Furye almashtirishini qo`llab, $F[\phi_0(x)]=\phi_0(p)$ bilgan holda quyidagini olamiz.

$$(2\pi ip)^4F[\bar{\psi}_\ell(x)]=(-1)^2\left(F[\varepsilon_{(0,1]}e^{2\pi i\omega x}]\phi_0(p)-\sum_{k=1}^{N}C_ke^{2\pi iphk}\phi_0(p)\right) \qquad (13)$$

(5) tenglikka asoslanib (13) formulaning o`ng tomonini nolga tenglaymiz. Shuning uchun (13) tenglamani ikkala tomonini $(2\pi ip)^4$ ga bo`lish mumkin. $F[\bar{\psi}_\ell]$ funksiyadan (13) quyidagicha aniqlanadi

$(-1)^2d_0\delta(p)$

U holda (13) quyidagicha bo`ladi

$$F[\bar{\psi}_\ell(x)]=(-1)^2d_0\delta(p)+\frac{F[\varepsilon_{(0,1]}e^{2\pi i\omega x}]\cdot\phi_0(p)}{(2\pi ip)^4}-\frac{\sum_{k=1}^{N}C_ke^{2\pi iphk}\cdot\phi_0(p)}{(2\pi ip)^4} \quad (14)$$

Demak, $\phi_0(p)$ qatorni δ - delta funksiyalarni xossalaridan foydalanamiz, ya'ni $\delta(x-a)f(x)=\delta(x-a)f(a)$

$$F[\bar{\psi}_\ell(x)]=(-1)^2d_0\delta(p)+F[\varepsilon_{(0,1]}e^{2\pi i\omega x}]\cdot\sum_{\beta\neq0}\frac{\delta(p-\beta)}{(2\pi i\beta)^4}-\sum_{k=1}^{N}C_ke^{2\pi iphk}\cdot\sum_{\beta\neq0}\frac{\delta(p-\beta)}{(2\pi i\beta)^4}$$

yoki

$$F[\bar{\psi}_\ell(x)]=(-1)^2d_0\delta(p)+F[\varepsilon_{(0,1]}e^{2\pi i\omega x}]\cdot\sum_{\beta\neq0}\frac{\delta(p-\beta)}{(2\pi i\beta)^4}-\sum_{k=1}^{N}C_k\cdot\sum_{\beta\neq0}\frac{e^{2\pi i\beta hk}\delta(p-\beta)}{(2\pi i\beta)^4}$$

Teskari Furye almashtirilishini qo`llab, oxirgi tenglik quyidagi ko`rinishni oladi

$$\bar{\psi}_\ell(x)=(-1)^2\left(d_0+\left(\varepsilon_{(0,1]}e^{2\pi i\omega x}\right)*\sum_{\beta\neq0}\frac{e^{-2\pi i\beta x}}{(2\pi i\beta)^4}-\sum_{k=1}^{N}C_k\cdot\sum_{\beta\neq0}\frac{e^{2\pi i\beta hk}e^{-2\pi i\beta x}}{(2\pi i\beta)^4}\right)$$

Bu yerdan

$$\bar{\psi}_{\ell}(x)=(-1)^2\left(d_0+\int_{-\infty}^{\infty}\left(\varepsilon_{(0,1]}e^{2\pi i\omega x}\right)\cdot\sum_{\beta\neq 0}\frac{e^{-2\pi i\beta(x-t)}}{(2\pi i\beta)^4}dt-\sum_{k=1}^{N}C_k\cdot\sum_{\beta\neq 0}\frac{e^{-2\pi i\beta(x-hk)}}{(2\pi i\beta)^4}\right)$$

quyidagi ko'rinishni yozib olamiz

$$\psi_{\ell}(x)=(-1)^2\left(\int_0^1 e^{-2\pi i\omega x}\cdot B_4(x-t)dt-\sum_{k=1}^{N}\bar{C}_k\cdot B_4(x-hk)+d_0\right) \quad (15)$$

$\ell(x)$ xatolik funksionali normasini hisoblaymiz.

Shunday qilib, (3) va (13) dan ushbu tenglikka ega bo`lamiz.

$$\|\ell\|^2=\left(\ell,\psi_{\ell}\right). \quad (16)$$

Avval $\ell(x)$ ni (3) ga asosan soddalashtiramiz

$$\ell(x)=\left(\varepsilon_{(0,1]}(x)\cdot e^{2\pi i\omega x}-\sum_{k=1}^{N}C_k\delta(x-hk)\right)*\phi_{\circ}(x)=$$

$$=\int_{-\infty}^{\infty}\left(\varepsilon_{(0,1]}(x-y)\cdot e^{2\pi i\omega(x-y)}-\sum_{k=1}^{N}C_k\delta(x-y-hk)\right)\cdot\sum_{\beta=-\infty}^{\infty}\delta(y-\beta)dy=$$

$$=\sum_{\beta=-\infty}^{\infty}\varepsilon_{(0,1]}(x-\beta)\cdot e^{2\pi i\omega(x-\beta)}-\sum_{k=1}^{N}C_k\sum_{\beta=-\infty}^{\infty}\delta(x-\beta-hk)=$$

$$=e^{2\pi i\omega x}\sum_{\beta=-\infty}^{\infty}\varepsilon_{(0,1]}(x-\beta)-\sum_{k=1}^{N}C_k\sum_{\beta=-\infty}^{\infty}\delta(x-\beta-hk)=$$

$$\ell(x)=e^{2\pi i\omega x}-\sum_{k=1}^{N}C_k\sum_{\beta=-\infty}^{\infty}\delta(x-\beta-hk) \quad . (17)$$

(12) va (17) ni inobatga olgan holda, (16) quyidagiga ega bo`lamiz.

$$\|\ell\|^2=\left(\ell,\psi_{\ell}\right)=\int_0^1\ell(x)\psi_{\ell}(x)dx=$$

$$=(-1)^2\int_0^1\left(e^{2\pi i\omega x}-\sum_{k=1}^{N}C_k\sum_{\beta=-\infty}^{\infty}\delta(x-\beta-h_k)\right)\cdot\left(\int_0^1 e^{-2\pi i\omega t}\cdot B_4(x-t)dt-\sum_{k'=1}^{N}\bar{C}_{k'}\cdot B_4(x-hk')\right)dx$$

$$=(-1)^2\left(\int_0^1 e^{2\pi i\omega x}e^{-2\pi i\omega t}B_4(x-t)dtdx-\sum_{k'=1}^{N}\bar{C}_{k'}\int_0^1 e^{2\pi i\omega x}\cdot B_4(x-hk')dx-\right.$$

$$-\sum_{k=1}^{N}C_k\int_0^1 e^{-2\pi i\omega t}\cdot\sum_{\beta=-\infty}^{\infty}\int_0^1\delta(x-hk-\beta)\cdot B_4(x-t)dxdt+$$

$$\left.+\sum_{k=1}^{N}\sum_{k'=1}^{N}C_k\bar{C}_{k'}\sum_{\beta=-\infty}^{\infty}\int_0^1\delta(x-hk-\beta)\cdot B_4(x-hk')dx\right)=$$

$$=(-1)^2\left(\int_0^1 e^{2\pi i\omega x}e^{-2\pi i\omega t}B_4(x-t)dtdx-\sum_{k'=1}^{N}\bar{C}_{k'}\int_0^1 e^{2\pi i\omega x}\cdot B_4(x-hk')dx-\right.$$

$$-\sum_{k=1}^{N} C_k \int_0^1 e^{-2\pi i\omega t} \cdot \sum_{\beta=-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(x-hk-\beta)\cdot \varepsilon_{(0,1]}(x)\cdot B_4(x-t)dxdt +$$

$$\left. +\sum_{k=1}^{N}\sum_{k'=1}^{N} C_k \bar{C}_{k'} \sum_{\beta=-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(x-hk-\beta)\cdot \varepsilon_{(0,1]}(x)\cdot B_4(x-hk')dx \right)$$

$$\|\ell\|^2 = (-1)^2 \left(\int_0^1\int_0^1 e^{2\pi i\omega x} e^{-2\pi i\omega t} B_4(x-t)dxdt - \sum_{k'=1}^{N} \bar{C}_{k'} \int_0^1 e^{2\pi i\omega x} \cdot B_4(x-hk')dx - \right.$$

$$-\sum_{k=1}^{N} C_k \int_0^1 e^{-2\pi i\omega t} \cdot \sum_{\beta=-\infty}^{\infty} \varepsilon_{(0,1]}(hk+\beta)\cdot B_4(hk+\beta-t)dt +$$

$$\left. +\sum_{k=1}^{N}\sum_{k'=1}^{N} C_k \bar{C}_{k'} \sum_{\beta=-\infty}^{\infty} \varepsilon_{(0,1]}(hk+\beta)\cdot B_4(hk+\beta-hk') \right) =$$

$$= (-1)^m \left(\int_0^1\int_0^1 e^{2\pi i\omega x} e^{-2\pi i\omega t} B_4(x-t)dxdt - \sum_{k'=1}^{N} \bar{C}_{k'} \int_0^1 e^{2\pi i\omega x} \cdot B_4(x-hk')dx - \right.$$

$$-\sum_{k=1}^{N} C_k \int_0^1 e^{-2\pi i\omega t} \cdot B_4(t-hk)dt \cdot \sum_{\beta=-\infty}^{\infty} \varepsilon_{(0,1]}(hk+\beta) +$$

$$\left. +\sum_{k=1}^{N}\sum_{k'=1}^{N} C_k \bar{C}_{k'} \cdot B_4(hk-hk') \cdot \sum_{\beta=-\infty}^{\infty} \varepsilon_{(0,1]}(hk+\beta) \right)$$

U holda xatolik funksionali normasi quyidagi ko‘rinishga ega bo’ladi.

$$\|\ell\|^2 = (-1)^2 \left(\sum_{k=1}^{N}\sum_{k'=1}^{N} C_k \bar{C}_{k'} \cdot B_4(hk-hk') - \sum_{k=1}^{N} \int_0^1 \left(\bar{C}_k \cdot e^{2\pi i\omega x} + C_k \cdot e^{-2\pi i\omega x} \right) \cdot B_4(x-hk)dx + \right.$$

$$+\int_0^1\int_0^1 e^{2\pi i\omega x} e^{-2\pi i\omega t} B_4(x-t)dxdt \qquad (18)$$

Adabiyotlar

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