



Umumlashgan Funksiyaning Butun Tartibli Hosilasi

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ABSTRACT

Maqolada umumlashgan funksiyalarning hosilasi, umumlashgan hosilaning xossalari keltirilgan va o‘rganilgan.

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Kirish

Umumlashgan funksiyalar bir qator qulay xossalarga egadir. Masalan, hosila tushunchasini tegishli ma’nodan umumlashtirish ixtiyoriy umumlashgan funksiyalarning cheksiz differensiallanuvchi bo‘lishligini ta’minlaydi va umumlashgan funksiyalardan tashkil topgan yaqinlashuvchi qatorni cheksiz marta hadma-had differensiallash mumkin bo‘lishligini bildiradi [1-7].

Umumlashgan funksiyaning hosilasi

$f(x) \in C^p(G)$ bo‘lgan funksiya bo‘lsin. U holda ixtiyoriy $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)$, $|\alpha| = \alpha_1 + \alpha_2 + \dots + \alpha_n \leq p$ multiindeks va ixtiyoriy $\varphi \in D(G)$ uchun

$$\begin{aligned} (D^\alpha f, \varphi) &= \int_G D^\alpha f(x) \varphi(x) dx = \\ &= (-1)^{|\alpha|} \int_G f(x) D^\alpha \varphi(x) dx = (-1)^{|\alpha|} (f, D^\alpha \varphi) \end{aligned}$$

bo‘laklab integrallash formulasi o‘rinli bo‘ladi. Bu tenglikni biz $f(x) \in D'(G)$ umumlashgan funksiyaning $D^\alpha f(x)$ (umumlashgan) hosilasining ta’rifi sifatida qabul qilamiz. Shunga ko‘ra, umumlashgan hosila ixtiyoriy $\varphi(x) \in D(G)$ uchun

$$(D^\alpha f, \varphi) = (-1)^{|\alpha|} (f, D^\alpha \varphi) \quad (1)$$

tenglik bilan kiritiladi.

Endi $D^\alpha f \in D'(G)$ umumlashgan funksiya bo'lishligini tekshiramiz. Haqiqatdan ham, agar $f(x) \in D'(G)$ bo'lsa, u holda $D^\alpha f(x)$ funksional (1) formulaning o'ng qismi bilan aniqlangan bo'lsa, u holda

$$\begin{aligned} (D^\alpha f, \lambda\varphi + \mu\psi) &= (-1)^{|\alpha|} (f, D^\alpha(\lambda\varphi + \mu\psi)) = \\ &= (-1)^{|\alpha|} (f, \lambda D^\alpha\varphi + \mu D^\alpha\psi) = \lambda(-1)^{|\alpha|} (f, D^\alpha\varphi) + \\ &+ \mu(-1)^{|\alpha|} (f, D^\alpha\psi) = \lambda(D^\alpha f, \varphi) + \mu(D^\alpha f, \psi) \end{aligned}$$

chiziqli bo'ladi. Shuningdek, agar $k \rightarrow \infty$ da D fazoda $\varphi_k \rightarrow 0$ yaqinlashuvchi bo'lsa, u holda

$$(D^\alpha f, \varphi_k) = (-1)^{|\alpha|} (f, D^\alpha\varphi_k) \rightarrow 0$$

yaqinlashuvchi ekanligini hosil qilamiz, ya'ni $D^\alpha f(x)$ funksional uzluksiz ham bo'ladi [8-19].

Xususan, $f = \delta$ umumlashgan funksiya uchun (1) tenglik ixtiyoriy $\varphi(x) \in D(G)$ uchun

$$(D^\alpha \delta, \varphi) = (-1)^{|\alpha|} D^\alpha \varphi(0)$$

shaklida bo'ladi.

Bu ta'rifdan, agar $f(x) \in D'(G)$ umumlashgan funksiya $G_1 \subset G$ bo'lgan ochiq to'plamda $C^p(G_1)$ sinfga qarashli bo'lsa, u holda $|\alpha| \leq p$ uchun $D^\alpha f(x)$ umumlashgan hosila va $\{D^\alpha f(x)\}$ klassik ma'nodagi hosilalar shu G_1 ochiq. to'plamda ustma-ust tushadi, ya'ni $|\alpha| \leq p$ va $x \in G_1$ uchun $D^\alpha f(x) = \{D^\alpha f(x)\}$ tenglik o'rinli bo'ladi.

Umumlashgan hosilaning xossalari

Umumlashgan funksiyalarni differensialash amali quyidagi xossalarga egadir:

a) $f \rightarrow D^\alpha f$ differensiallash amali $D'(G)$ fazoni $D'(G)$ fazoga akslantiruvchi chiziqli va uzluksiz operator bo'ladi. Shunga ko'ra, ixtiyoriy $f, g \in D'(G)$ umumlashgan funksiya uchun

$$D^\alpha(\lambda f + \mu g) = \lambda D^\alpha f + \mu D^\alpha g$$

tenglik o'rinli bo'ladi. Bu chiziqlilik xossasini isbot qilamiz. Haqiqatdan ham ixtiyoriy $\varphi(x) \in D(G)$ uchun

$$\begin{aligned} (D^\alpha(\lambda f + \mu g), \varphi) &= (-1)^{|\alpha|} (\lambda f + \mu g, D^\alpha\varphi) = \\ &= \lambda(-1)^{|\alpha|} (f, D^\alpha\varphi) + \mu(-1)^{|\alpha|} (g, D^\alpha\varphi) = \\ &= \lambda(D^\alpha f, \varphi) + \mu(D^\alpha g, \varphi) \end{aligned}$$

tenglik o'rinli bo'ladi.

Agar $k \rightarrow \infty$ da $D'(G)$ fazoda $f_k \rightarrow 0$ yaqinlashuvchi bo'lsa, u holda $k \rightarrow \infty$ da $D'(G)$ fazoda $D^\alpha f_k \rightarrow 0$ yaqinlashuvchi bo'ladi. Bu uzluksizlik xossasini ham isbot qilamiz. $k \rightarrow \infty$ da $D'(G)$ fazoda $f_k \rightarrow 0$ yaqinlashuvchi bo'lsin. u holda ixtiyoriy $\varphi(x) \in D(G)$ uchun $k \rightarrow \infty$ da

$$(D^\alpha f_k, \varphi) = (-1)^{|\alpha|} (f_k, D^\alpha \varphi) \rightarrow 0$$

yaqinlashuvchi bo'ladi. Bu esa $k \rightarrow \infty$ da $D'(G)$ fazoda $D^\alpha f_k \rightarrow 0$ yaqinlashuvchi bo'lishligini bildiradi.

Masalan, $\varepsilon \rightarrow +0$ da $D'(R^n)$ fazoda

$$D^\alpha \omega_\varepsilon(x) \rightarrow D^\alpha \delta(x) \tag{2}$$

yaqinlashuvchi bo'ladi.

b) Ixtiyoriy $f(x) \in D'(G)$. umumlashgan funksiya (xususan, G ochiq to'pamda lokal jamlanuvchi ixtiyoriy funksiya) (umumlashgan ma'noda) cheksiz differensiallanuvchi bo'ladi.

Haqiqatdan ham, agar $f(x) \in D'(G)$ bo'lsa, u holda $\frac{\partial f}{\partial x_i}(x) \in D'(G)$ bo'ladi. Xuddi shuningdek, o'z

navbatida $\frac{\partial}{\partial x_j} \left(\frac{\partial f}{\partial x_i} \right) \in D'(G)$ bo'ladi va xokazo.

v) Differensiallashning natijasi differensiallashning tartibiga bog'lik emas.

Masalan: ixtiyoriy $f(x) \in D'(G)$ uchun

$$D_1(D_2 f) = D_2(D_1 f) = D^{(1,1)} f \tag{3}$$

tenglik o'rinli bo'ladi.

Haqiqatan ham, ixtiyoriy $\varphi(x) \in D(G)$ uchun

$$(D^{(1,1)} f, \varphi) = (f, D_2 D_1 \varphi) = (D_1(D_2 f), \varphi) = (D_2(D_1 f), \varphi)$$

tenglik o'rinli bo'ladi.

Umuman olganda

$$D^{\alpha+\beta} f = D^\alpha (D^\beta f) = D^\beta (D^\alpha f) \tag{4}$$

tenglik o'rinlidir.

Haqiqatan ham, ixtiyoriy $\varphi(x) \in D(G)$ uchun

$$\begin{aligned} (D^{\alpha+\beta} f, \varphi) &= (-1)^{|\alpha|+|\beta|} (f, D^{\alpha+\beta} \varphi) = (-1)^{|\alpha|} (D^\beta f, D^\alpha \varphi) = \\ &= (D^\alpha (D^\beta f), \varphi) = (-1)^{|\beta|} (D^\alpha f, D^\beta \varphi) = (D^\beta (D^\alpha f), \varphi) \end{aligned}$$

tenglik o‘rinli va bundan esa (4) tenglik kelib chiqadi

g) Agar $f(x) \in D'(R^n)$ va $a(x) \in C^\infty(R^n)$ bo‘lsa, u holda $a(x)f(x)$ ko‘paytma funksiyani differensiallash uchun

$$D^\alpha (af) = \sum_{\beta \leq \alpha} \binom{\beta}{\alpha} D^\beta a D^{\alpha-\beta} f \quad (5)$$

Leybnis formulasi o‘rinli bo‘ladi.

Haqiqatdan ham, agar ixtiyoriy $\varphi(x) \in D(G)$ funksiya bo‘lsa, u holda

$$\begin{aligned} \left(\frac{\partial(a(x)f(x))}{\partial x_1}, \varphi \right) &= - \left(a(x)f(x), \frac{\partial \varphi}{\partial x_1} \right) = - \left(f(x), a(x) \frac{\partial \varphi}{\partial x_1} \right) = \\ &= - \left(f(x), \frac{\partial(a(x)\varphi)}{\partial x_1} - \frac{\partial a(x)}{\partial x_1} \varphi \right) = - \left(f(x), \frac{\partial(a(x)\varphi)}{\partial x_1} \right) + \\ &+ \left(f(x), \frac{\partial a(x)}{\partial x_1} \varphi \right) = \left(\frac{\partial f(x)}{\partial x_1}, a(x)\varphi \right) + \left(\frac{\partial a(x)}{\partial x_1} f(x), \varphi \right) = \\ &= \left(a(x) \frac{\partial f(x)}{\partial x_1}, \varphi \right) + \left(\frac{\partial a(x)}{\partial x_1} f(x), \varphi \right) = \left(a(x) \frac{\partial f}{\partial x_1} + \frac{\partial a(x)}{\partial x_1} f, \varphi \right) \end{aligned}$$

tenglik o‘rinli va bundan (5) tenglik $\alpha = (1, 0, \dots, 0)$ uchun kelib chiqadi. Shunga ko‘ra, matematik induksiya usulini ko‘llab, biz (5) formulani ixtiyoriy $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)$ multiindeks uchun isbot qilishimiz mumkin bo‘ladi [20-34].

d) Agar $x \in G$ ochiq to‘plamda umumlashgan funksiya $f = 0$ teng bo‘lsa, u holda shu $x \in G$ ochiq to‘plamda $D^\alpha f = 0$ tenglik o‘rinli bo‘ladi, shunga ko‘ra $\text{supp } D^\alpha f \subset \text{supp } f$ munosabat o‘rinli bo‘ladi.

Haqiqatdan ham, agar $\varphi(x) \in D(G)$ bo‘lsa, u holda $D^\alpha \varphi(x) \in D(G)$ bo‘ladi. Shunga ko‘ra ixtiyoriy $\varphi(x) \in D(G)$ funksiya uchun

$$(D^\alpha f, \varphi) = (-1)^{|\alpha|} (f, D^\alpha \varphi) = 0 \quad (6)$$

tenglik o‘rinli bo‘ladi va bu tenglik $x \in G$ uchun $D^\alpha f = 0$ ekanligini bildiradi.

u) Agar lokal integrallanuvchi ik $u_k(x)$ funksiyalardan tuzilgan

$$\sum_{k=1}^x u_k(x) = S(x)$$

qator har bir kompaktda tekis yaqinlashuvchi bo'la, u holda bu qatorni istalgan marta hadma-had differensiallash mumkin va differensiallashdan hosil qilingan qatorlar $D'(R^n)$ fazoda yaqinlashuvchi bo'ladi [35-41].

Haqiqatdan ham, ixtiyoriy $R > 0$ uchun $p \rightarrow \infty$ da

$$S_p(x) = \sum_{k=1}^p u_k(x) \Rightarrow S(x)$$

tekis yaqinlashuvchi bo'ladi. Shunga ko'ra $p \rightarrow \infty$ da $D'(R^n)$ fazoda $S_p \rightarrow S$ yaqinlashuvchi bo'ladi.

Yuqoridagi a) xossaga ko'ra $p \rightarrow \infty$ da $D'(R^n)$ fazoda

$$D^\alpha S_p(x) = \sum_{k=1}^p D^\alpha u_k(x) \rightarrow D^\alpha S(x)$$

yaqinlashuvchi bo'ladi. Bu esa xossaning tasdig'ini bildiradi [39-45].

Ushbu xossadan quyidagi xulosa kelib chiqadi: agar

$$|a_k| \leq A |k|^m + B \quad (6)$$

tengsizlik o'rinli bo'lsa, u holda

$$\sum_{k=-\infty}^{\infty} a_k e^{ikx}$$

trigonometrik qator $D'(R^1)$ fazoda yaqinlashuvchi bo'ladi.

Haqiqatdan ham, (3.6) munosabataga ko'ra

$$\frac{a_0 x^{m+2}}{(m+2)!} + \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \frac{a_k}{(ik)^{m+2}} e^{ikx} \quad (7)$$

qator R^1 fazoda tekis yaqinlashuvchidir. Shuning uchun uning $m+2$ tartibli hosilasi $D'(R^1)$ fazoda yaqinlashuvchi bo'lib, bu qator esa (7) qator bilan ustma-ust tushadi.

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