

Optimization Of Sprinkler Spacing Under Wind Influence: Mathematical Modeling And Analysis

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ABSTRACT

This article analyzes, based on theoretical modeling, the influence of wind speed and water droplet diameter on the coverage distance in sprinkler irrigation systems. The trajectory of a water droplet is expressed through mathematical equations that account for aerodynamic forces and wind effects. Based on the obtained results, the dependence of the center-to-center distance between sprinklers (S) on wind speed and droplet diameter was analyzed graphically. The results showed that as wind speed increases, the coverage distance decreases significantly, and smaller droplets tend to deviate more. Considering that the average wind speed in Uzbekistan ranges from 2–5 m/s, it was determined that the optimal distance between sprinklers should be in the range of 8–12 meters. The research findings have practical significance for the design and efficiency improvement of sprinkler irrigation systems.

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Introduction. Under conditions of limited water resources in agriculture, the implementation of efficient irrigation technologies is of great importance. Sprinkler irrigation systems (sprinklers, central pivots, stationary sprinkler networks) allow for uniform and controlled water distribution while helping to reduce water consumption [1]. However, in practice, under untested or open-field conditions, wind effects can lead to water loss due to droplet drift and evaporation, reaching up to 24–40% [2]. The influence of wind significantly affects the flight trajectory of water droplets and their distribution within the controlled irrigation area (Figure 1). Under wind influence, the uniformity of irrigation distribution decreases [3]. Moreover, the droplet's spreading angle, velocity, and geometry (diameter) are also factors that amplify the wind's impact [4]. For instance, droplets with smaller diameters are more sensitive to wind, showing greater deviations in their trajectories.

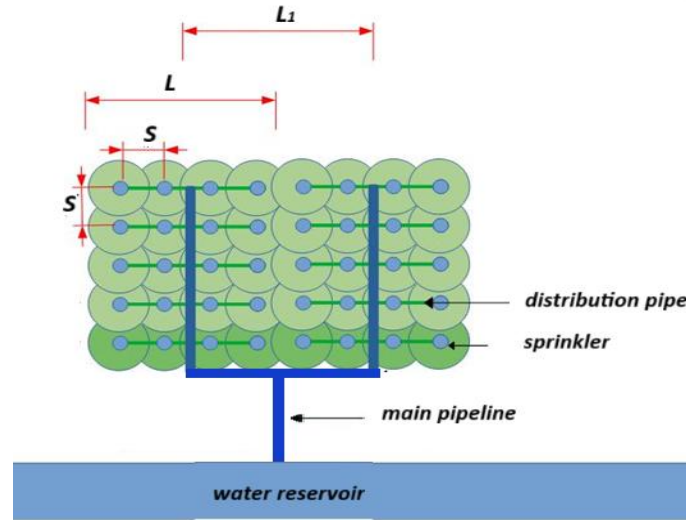


Fig. 1. Schematic diagram of a sprinkler irrigation system: S – distance between sprinklers; L – coverage width of sprinklers connected to a single distribution pipe; L_1 – distance between adjacent distribution pipes.

Therefore, it is necessary to analyze the motion of a water droplet under wind influence using mathematical models. These models include the effects of gravitational forces, aerodynamics, and wind forces (including wind direction and relative velocity). In this paper, the trajectory of a droplet released from a stationary sprinkler under wind influence is analyzed based on mathematical equations, modeled using discrete integration methods, and the results are discussed.

Materials and Methods

The motion of a water droplet ejected from a sprinkler in the air is primarily determined by gravitational force, aerodynamic drag, wind influence, and the droplet's mass. In addition, droplet diameter and air conditions (temperature, humidity, density) are important parameters of the model.

The description of droplet motion in air is based on **Newton's second law**. The forces acting on the droplet include:

Gravitational force:

$$F_g = mg = \frac{\pi}{6} d_s^3 \rho_c g, \quad (1)$$

Aerodynamic drag force (in Stokes and turbulent regimes):

$$F_d = \frac{1}{2} \rho_m C_x A (\vartheta - \vartheta_{sh})^2, \quad (2)$$

where $A = \frac{\pi d_s^2}{4}$ is the projected area of the droplet.

Wind effect: the horizontal wind velocity (ϑ_{sh}) causes the droplet's trajectory to deviate from its original path.

Based on this, the motion of the droplet in the x and y coordinates is described by differential equations. In determining the trajectory, aerodynamic forces, air resistance, and wind influence were taken into account.

The droplet dynamics are modeled using differential equations, and the resistance coefficient of the medium is expressed as [5]:

$$K_1(t) = - \left(\frac{18\mu}{\rho_c d_s^2} + \frac{\rho_m C_x \left(\sqrt{\vartheta_x^2(t) + \vartheta_y^2(t)} - \vartheta_{sh} \sin \theta \right)}{4 \rho_s d_s} \right). \quad (3)$$

The initial velocity of the droplet, considering wind influence, is given by:

$$\vartheta_x(0) = |\vec{\vartheta}(0)| \cos \alpha - \vartheta_{sh} \sin \theta; \quad (4)$$

$$\vartheta_Y(0) = |\vec{\vartheta}(0)| \sin \alpha. \quad (5)$$

The droplet trajectory is calculated step by step using the discrete integration method [6]:

$$x(t_{i+1}) = (1 + K_1(t_i)\Delta t)\vartheta_x(t_i); \quad (6)$$

$$y(t_{i+1}) = \left(\frac{\rho_c}{\rho_t} - 1\right) \cdot 9,81 \cdot \Delta t + (1 + K_1(t_i) \cdot \Delta t \cdot \vartheta_y(t_i)). \quad (7)$$

here μ — dynamic viscosity of air, Pa·s; d_s — droplet diameter, m; ρ_c — water density, kg/m³; ρ_m — air density, kg/m³; C_x — air drag coefficient; ϑ_{sh} — wind speed, m/s; θ — wind direction angle, degrees.

Results and Discussion.

Using the mathematical model, simulations were performed for various wind speeds ($\vartheta_{sh} = 0, 2, 4, 6$ m/s) and droplet diameters ($d_s = 0.5, 1.0, 1.5$ mm). The results are shown graphically in Fig. 2.

From the results, it can be seen that larger droplets are less affected by wind, so their trajectories remain almost linear. Smaller droplets, however, deviate more under wind influence, and their flight distance decreases significantly. At wind speeds of $\vartheta_{sh} = 4-6$ m/s, the uniformity of irrigation distribution is disrupted, requiring additional hydraulic adjustments to ensure even coverage [7, 8].

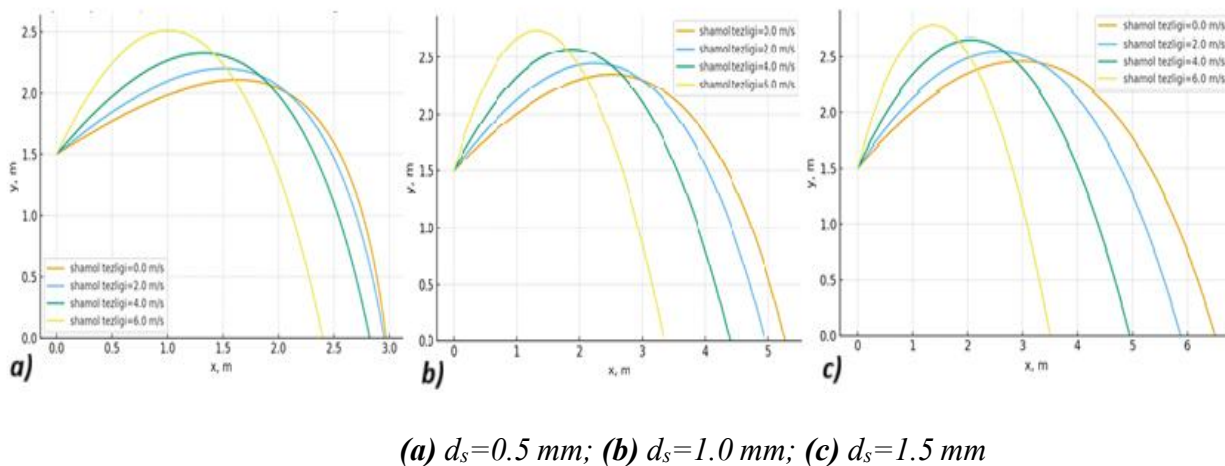


Fig. 2. Variation of the water droplet's flight trajectory depending on wind speed ($\alpha=30^\circ$, $\vartheta_o=5$ m/s).

Graphical analyses show that a droplet with $d_s=0.5$ mm deviates by nearly 30–40% under wind speed $\vartheta_{sh}=6$ m/s, while a droplet with $d_s=1.5$ mm almost maintains its original trajectory.

Based on the trajectory graphs obtained from the analysis of droplet motion under wind influence, the optimal spacing between two sprinklers was determined. These calculations take into account different wind speeds and the sprinkler coverage radius (Fig. 3).

In most regions of Uzbekistan, the average wind speed ranges from $\vartheta_{sh} = 2-6$ m/s, and in some cases can reach $\vartheta_{sh} = 8-10$ m/s. Therefore, this wind speed range was used as the basis for the model.

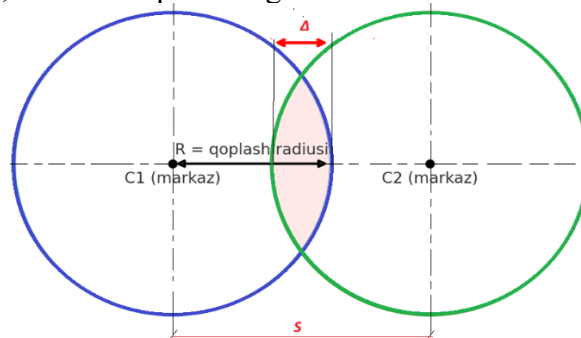


Fig. 3. Diagram for determining the spacing between sprinklers:

R — sprinkler coverage radius (m); Δ — overlap distance between sprinklers (m); S — center-to-center spacing between sprinklers (m).

If the overlap is equal to Δ :

$$S = 2R - \Delta. \quad (8)$$

The intersection area A_{ov} between two circles with the same radius R (for $0 \leq S \leq 2R$) is given by:

$$A_{ov} = 2R^2 \cos^{-1} \left(\frac{S}{2R} \right) - \frac{S}{2} \sqrt{4R^2 - S^2}. \quad (9)$$

If $S \geq 2R$, the circles do not overlap $A_{ov}=0$

if $S \leq 0$, one circle completely covers the other, and $A_{ov} = \pi R^2$.

The total area of a single circle is:

$$A_{tot} = \pi R^2. \quad (10)$$

The percentage of overlap (i.e., the ratio of the overlapping area of two circles to the total area of one circle) is calculated as:

$$\text{Overlap, \%} = \frac{A_{ov}}{A_{tot}} \cdot 100\%. \quad (11)$$

By analyzing droplet trajectory graphs under different wind speeds, this relationship allows the optimal center-to-center spacing S between sprinklers to be determined for various wind conditions

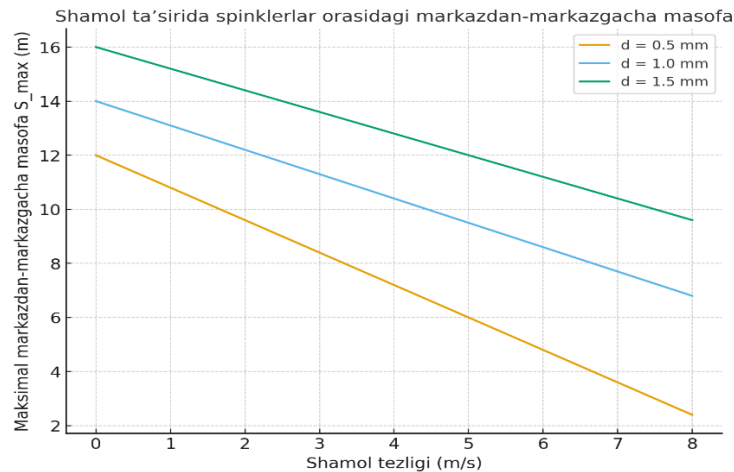


Fig. 4. Effect of wind speed on the spacing between sprinklers

The analysis of the graphs in Fig. 5 shows that as wind speed increases, the maximum spacing $S_{\max}$ between sprinklers decreases sharply. For droplet diameter $d_s=0.5$ mm, when wind speed ϑ_{sh} increases from 0 to 8 m/s, the spacing S drops from approximately 12 m to 0 m.

Larger droplets ($d_s=1.5$ mm) are less sensitive to wind: even at $\vartheta_{sh}=8$ m/s, the spacing S remains around 8–9 m. Increasing the droplet diameter helps maintain the stability of water distribution, whereas smaller droplets are more easily carried away by the wind, creating gaps in coverage.

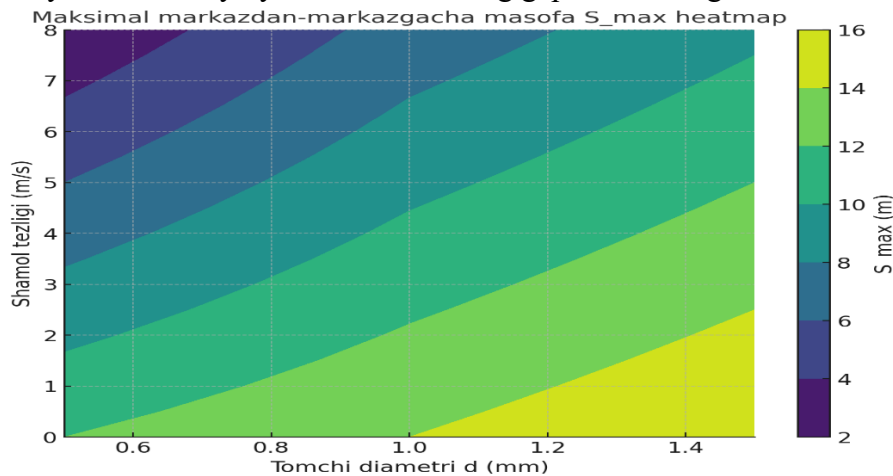


Fig. 5. Effect of droplet diameter on the spacing between sprinklers

For conditions in Uzbekistan, it is advisable to consider the average wind speed within the range of $\vartheta_{sh}=2\text{--}5$ m/s for design calculations. If the sprinkler system operates with small droplets ($d_s=0.5$ mm), at wind speeds of $\vartheta_{sh}=3\text{--}4$ m/s, the coverage distance decreases to about 5–7 m, leading to a potential loss in irrigation efficiency. Larger droplets ($d_s=1.0\text{--}1.5$ mm) are more resistant to wind effects; even at $\vartheta_{sh}=5\text{--}6$ m/s, the coverage distance S remains around 8–12 m. Therefore, in practice, the spacing between sprinklers should always be selected in accordance with both wind speed and droplet diameter.

Conclusion. This study analyzed the effect of wind speed and droplet diameter on the sprinkler spacing in rainfall irrigation systems through mathematical modeling.

The results indicate the following:

As wind speed increases, the maximum center-to-center spacing S_{max} between sprinklers decreases significantly.

1. Small droplets ($d_s=0.5$ mm) are highly sensitive to wind; when ϑ_{sh} exceeds 4–5 m/s, the irrigation uniformity decreases sharply.
2. Larger droplets ($d_s=1.0\text{--}1.5$ mm) are more stable, maintaining satisfactory coverage even under wind speeds of 6–7 m/s.
3. Considering the average wind speed in Uzbekistan ($\vartheta_{sh}=2\text{--}5$ m/s), this factor plays a decisive role in selecting the optimal sprinkler spacing.

To ensure efficient use of sprinkler irrigation systems in Uzbekistan, wind speed and droplet diameter must be carefully accounted for when positioning sprinklers. Given that average wind speeds typically range between 2–5 m/s, the center-to-center spacing between sprinklers should not exceed 8–12 m. Additionally, using sprinklers that produce larger droplets instead of smaller ones enhances coverage stability and ensures more uniform water distribution.

The obtained results provide a practical methodological basis for designing sprinkler irrigation systems, selecting appropriate sprinkler types, and determining optimal field placement under local climatic conditions.

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